Physics Extended Essay

How can the behavior of a weather balloon in the Earths atmosphere be described with physics?

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Contents

Abstract

High Altitude Balloons (HABs) are tools used by hobbyists and scientists alike to gain access to high altitudes. Currently, these balloons are simply released into the air and left to the forces of nature. This essay seeks to understand the behavior of HABs in the Earth's atmosphere by considering all factors and forces that affect its movement. By evaluating and analyzing all factors, I hypothesize that I will be able to form an holistic equation to predict the behavior of a balloon and match it with experimental data.

The essay attempted to use fundamental physical laws and theorems to describe the effects of wind, temperature, radiation, buoyancy, rubber elasticity, and gravity on the balloon. These were then all combined into a single equation. This equation was then implemented into a MATLAB script, which was then able to successfully model the vertical behavior of a balloon. Combined with a particle path prediction tool from the NOAA, a complete prediction for the balloon's behavior was formed.

Actual flight data was collected by a GPS unit on a weather balloon launched in August, 2013. A comparison with the modeled prediction of that flight showed a very close correlation, verifying the integrity of the prediction. In conclusion, I was able to form a valid holistic equation to predict the behavior of a balloon by considering all factors.

1. Introduction

High Altitude Balloons, or HABs, are latex weather balloons used to transport small payloads to very high altitudes for short periods of time. These balloons are filled with light gases (Helium and Hydrogen are often used) and let go into the air. These balloons can go up as high as 30000m and above, and serve as a relatively cheap method of accessing high altitudes. As the balloon rises, it expands dramatically. Once its material reaches its stretching limit, the balloon bursts and the payloads come tumbling back to earth. The balloons can also reach neutral buoyancy, hovering in the air in equilibrium. These balloons can last for several days, eventually bursting due to solar radiation.

These balloons are important due to their usefulness and affordability. These balloons can go up to higher altitudes than spy planes for a fraction of the cost. They can be launched by anyone from students to experienced researchers. They are used in various scientific and commercial fields, and payloads can include scientific equipment, cameras, and other devices. Most importantly, the provide access to a region seldom visited; near-space. Planes can travel below this region, and rockets can travel to above it. However, only weather balloons can reach and remain in the mesosphere.

During one of many balloon launches by my balooning group, I asked myself, “How can the behavior of a weather balloon in the Earth’s atmosphere be described with physics?” In this essay, I am seeking to clearly define and model the behavior of high altitude balloons by considering the various environmental and material factors that affect it. By creating a scientific understanding of such behavior, I am hoping to provide a better resources for amateur, professional, and scientific ballooning groups. A model that describes all factors that affect a balloon’s behavior would be beneficial to everyone that uses these balloons, at it would provide a scientific foundation to an object previously left to the forces of nature.

Several papers have already explored the use of high altitudes balloons in an educational environment (Flaten, 2013; Larson et al., 2009). However, they describe student involvement and the process of launching a weather balloon, not the behavior of the balloon istelf. Other papers and books have discussed the material properties of a rubber shell (Kanner, 2007; Gent, 1999). There have also been other papers that have attempted to predict the behavior of weather balloons (Conner and Arena, 2010; Dai et al., 2012). However, these papers are either too complex for practical use by amateurs or fail to take into account several factors. My paper will attempt to describe the behavior simply while still covering as many factors as possible.

2. Horizontal Motion

2.1. Theory

Although drafts and thermals may affect vertical movement, they only cause temporary displacement and occur mainly at low altitudes. Air currents, or jet streams, have a much larger effect on HABs by causing a large amount of lateral displacement throughout the flight in all altitudes.

The wind causes lateral displacement in an HAB through wind resistance, or drag. The spherical balloon can reach diameters up to approximately 8 meters, resulting in a large amount of drag

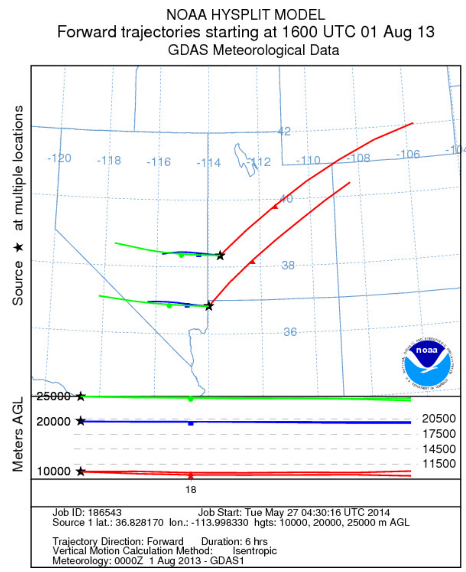

Figure 1: Predicted trajectory for a particle at 10000m (red), 20000m (blue), and 30000m (green)

from the wind. The wind velocity is constantly changing at different locations and times, and can only be predicted up to a few days in the future. The HYSPLIT model by the National Oceanic and Atmospheric Association provides the best model for these predictions (NOAA, 2014).

2.2. Trajectory Modeling

This section will demonstrate the use of the HYSPLIT model in predicting the trajectory of a weather balloon. Figure 1 shows the trajectory for the FAST-12, predicted by the HYSPLIT model at different altitudes. However, since the balloon is constantly rising, the resultant trajectory would be a combination of the different colored trajectories. More specifically, it would travel on the red path at first and turn into the blue/green path as it rises. Figure 2 clearly displays the similarity between the predicted trajectory and the actual trajectory of the balloon.

Thus, it has been shown through experimentation that by using both the HYSPLIT model and the lift velocity/altitude model displayed above, the behavior of an HAB in the Earth's atmosphere can be displayed and predicted with some accuracy.

3. Basics of Vertical Motion

The basics of the vertical motion of a weather balloon can be described by this equation:

$$\Sigma F = F_B - F_g = F_D \tag{3.1}$$

A launched balloon's net lift force is composed of two vertical forces; the upwards buoyant force, F_B and the gravitational force, F_g. The vertical drag force, or F_D, is a result of the net lift force,

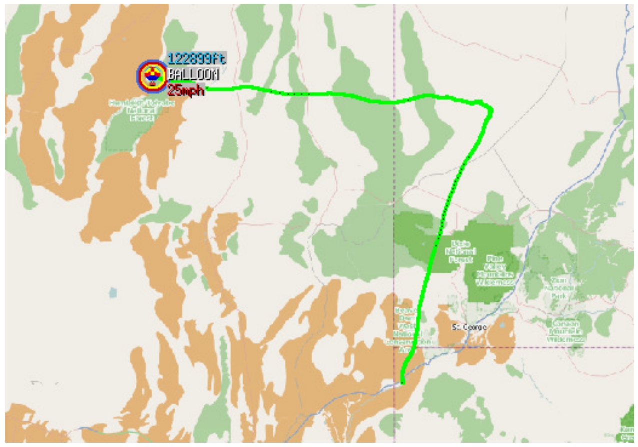

Figure 2: Actual trajectory of the weather balloon

and later becomes useful in calculating ascent velocity. Each of these forces will now be considered separately.

3.1. Gravitational Force

Gravitational force is the easiest to calculate. All aspects of the balloon's mass, including the payload, the balloon itself, and the gas inside of the balloon, need to be measured so that Newton's Second Law can be applied.

$$F_g = mg \tag{3.2}$$

where m is the mass and g is the gravitational acceleration, or $9.8 m/s^2$

No further calculations or measurements are necessary, unless the balloon is using a ballast system by dropping weights mid-flight. However, for the sake of our model, it will be assumed that the balloon stays at a constant mass throughout its flight.

3.2. Drag Force

The drag force is most useful for calculating the ascent velocity of the balloon. By assuming that the balloon is always rising at terminal velocity during all points of its flight, it can also be assumed that the net force acting upon the balloon is zero, resulting in this equation:

$$F_B - F_g - F_D = 0 \tag{3.3}$$

The drag force on a sphere (F_D) can be described as such:

$$F_D = \frac{1}{2}\rho v^2 C_d A \tag{3.4}$$

where ρ is the density of the fluid (which can be found through the US Standard Atmosphere (NOAA, 1976)), v is the velocity of the balloon, C_d is the drag coefficient, and A is the cross-sectional area. Note that many of these values change with altitude.

By isolating v (all other values can be measured or calculated), the theoretical vertical velocity of the balloon can be calculated and ultimately compared to experimental data. This method will be used extensively in section 6.2 to verify the validity of this model.

3.3. Buoyant Force

When a balloon is launched, the dominant buoyant force causes the balloon to accelerate upwards. The fundamentals of buoyancy can be calculated using the Archimedes Principle, which is:

$$F_B = \rho(A)Vg \tag{3.5}$$

where $\rho(A)$ is the atmospheric density function with respect to altitude, V is the volume of the balloon, and g is the gravitational acceleration.

The density is a function of altitude, and can be found in the US Standard Atmosphere (NOAA, 1976). The gravitational acceleration is $9.8m/s^2$. Balloon volume is harder to calculate, as it is dynamic throughout flight.

The volume of an HAB increases significantly throughout its trip, potentially increasing tenfold throughout its flight (Maxham et al., 2014). This expansion is a significant factor to the buoyant force of the balloon.

The volume of an HAB relies on various factors: the gas, temperature, and pressure. By using the ideal gas law, all of these factors can be considered to calculate the volume of the gas.

$$PV = nRT \tag{3.6}$$

where n is the number of moles of gas (which can be measured with a flowmeter), R is the universal gas constant (8.314), T is the temperature in Kelvin, and P is the pressure in pascals.

However, experimentation shows clearly that the ideal gas law alone is unable to adequately describe the behavior of an HAB in the atmosphere. Therefore, several factors are included in order to fully understand this behavior and accurately reflect the experimental data.

4. Further Exploration

4.1. Variations in Drag with Altitude

4.1.1. Variations in the Drag Coefficient

The drag coefficient for a sphere has a direct relationship with the Reynolds number. The Reynold's number is a constant used in fluid mechanics to predict flow patterns in viscous fluids, and is especially important for calculating drag coefficients for curved objects. Since the balloon is a sphere, this number is important to the calculations. The US Standard Atmosphere provides the Reynold's number for specific altitudes.

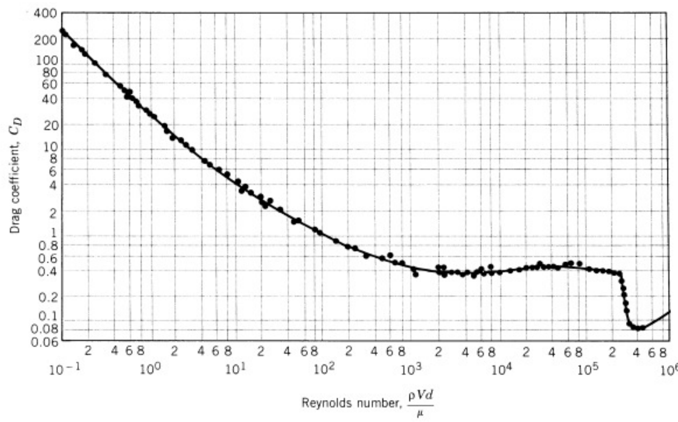

Figure 3: Drag Coefficient vs. Reynold's Number for a Sphere (Cimbala, 2012)

Figure 3 shows the relationship between the drag coefficient and the Reynold's number. By using it in conjunction with the data provided by the US Standard Atmosphere, it is possible to calculate the drag coefficient for the balloon at most altitudes.

The equation for the drag force on the balloon finally takes this form:

$$F_D = \frac{1}{2} d(A) v^2 c(R(A)) \pi r^2 \tag{4.1}$$

where $d(A)$is the density with respect to altitude, $c(R(A))$ is the drag coefficient with respect to the Reynolds number, which changes with altitude, v is the velocity of the balloon, and r is the radius of the balloon.

4.1.2. Variations in the Cross-Sectional Area

Note that the radius of the balloon varies with altitude, so the cross-sectional area will also vary. The variation of the balloon's volume will be discussed in section 4.3; the radius can then be calculated from the volume by using:

$$r = \sqrt[3]{\frac{3V}{4\pi}} \tag{4.2}$$

and

$$A = \pi r^2 \tag{4.3}$$

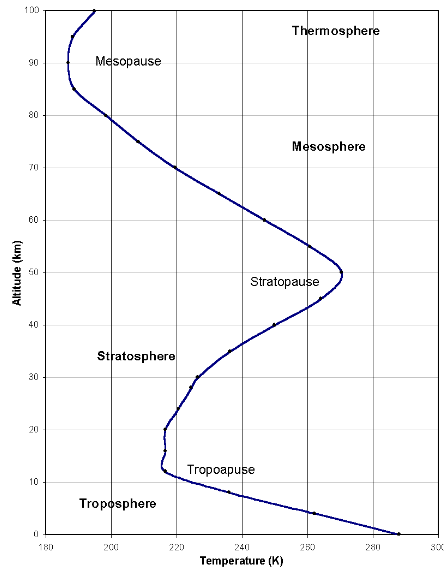

Figure 4: Altitude vs. Temperature provided by the US Standard Atmosphere

where V is the volume of the baloon in m^3, r is the radius of the balloon in m, and A is the cross-sectional area of the balloon in m^2.

4.2. Variations in Temperature

4.2.1. Atmospheric Temperature

Temperature has a direct impact on the vertical movement of a balloon. As the balloon heats up or cools down, its volume increases or decreases, which affects its buoyant force, as buoyancy is dependant on volume (as shown in section 3.3).

The US Standard Atmosphere provides a base for this investigation (NOAA, 1976). It provides average temperatures for specific altitudes in the Earths atmosphere, and the graph with this data is shown in figure 4. However, other factors also need to be taken into account.

First, diurnal temperature variation, or the temperature change between day and night, needs to be considered. Linacre suggests that the range of temperature variation is minimal for altitudes 750 meters and higher (Linacre, 1982). Since the HAB spends most of its time at this range of altitudes, diurnal temperature variation may be considered insubstantial, in terms of atmospheric temperature; diurnal radiation from the sun can play a significant role on the temperature of the balloon (discussed in 4.2.2).

Second, seasonal temperature variation, or the temperature change between seasons, also needs to be considered. Gerding provides temperature data at 54 °N from a LIDAR (a remote sensing method that uses light instead of sound), suggesting temperature variations up to 27 Kelvin. However, this variation changes every year for different latitudes and alitudes (Gerding et al., 2008). As there are no methods of mathematically predicting these variations, this factor must be taken into account manually on a case-by-case basis. Therefore, it will simply be represented as a constant, ΔT_s, within the model.

4.2.2. Solar Radiation

Radiation may play a larger role in changing the temperature of the gas inside the balloon, as the balloon is often exposed to the sun for extended periods of time.S olar radiation changes the temperature of the balloon by radiating energy directly at the balloon membrane. Solar radiation is drastically increased above the ozone layer, in which an HAB spends a lot of its time. Although the radiation only exists during the day, it is still a critical factor to consider.

Solar radiation has an intensity of about $1400Wm^{-2}$ (Tsokos, 2008). The power delivered to the balloon can be calculated through

$$P = IA \tag{4.4}$$

Where P is power in watts, I is intensity in Wm^{-2}, and A is the cross-sectional area in m^2. The cross-sectional area was mentioned previously when discussing drag force in section 3.2; the same value is used here.

To calculate the temperature change that this radiation has on this balloon, this equation can be used:

$$Q = mc\Delta T_r \tag{4.5}$$

where Q is the heat added in joules, m is the mass of gas, c is the specific heat capacity of the gas, and ΔT_r is the change in temperature due to radiation.

Since heat is defined as the power multiplied by time,

$$Pt = mc\Delta T_r \tag{4.6}$$

where P is the power in watts and t is the time in seconds. When solved for ΔT_r, the increase in temperature per time due to radiation can be found.

However, note that the rubber balloon is not a perfect absorber; it only absorbs a portion of the total incident radiation received. Gray soft rubber has an absorption factor of 0.65, which means it absorbs 65% of radiation received (TheEngineeringToolbox, 2000). However, since the rubber used in weather balloons is white, it can be assumed that the absoroption factor is lower. The difference in absorption factor between white and gray materials is 0.08 (TheEngineeringToolbox, 2000), so it can be calculated that the absorption factor of a white rubber surface is 0.57.

Applying this to the original heat transfer equation,

$$0.57Pt = mc\Delta T_r \tag{4.7}$$

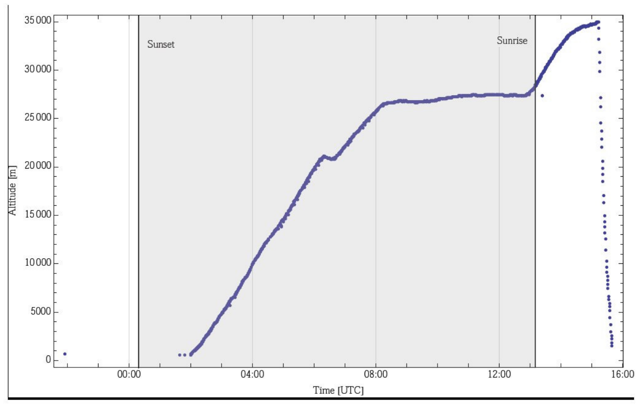

Figure 5: An altitude graph showing the signifcant effects of solar radiation on the buoyancy of a weather balloon.

or

$$\Delta T_r = \frac{0.57Pt}{mc} \tag{4.8}$$

Equation 4.8 is the final representation of the heat gained by the balloon due to solar radiation.

4.3. Elastic Pressure

Pressure, however, is more complicated. Taking the atmospheric pressure from the US Standard Atmosphere is not adequate; pressure from the elastic material of the balloon must also be taken into account. Since the internal pressure (P_B) of the balloon must equal the atmospheric pressure (P_{atm}) and the elastic pressure ($P_{elastic}$) of the balloon, the relationship can be described as such:

$$P_B = P_{atm} + P_{elastic} \tag{4.9}$$

Rubber resistance is a very complicated subject. Multiple models exist to represent the resistance of a spherical rubber shell. One of these is the Mooney-Rivlin model (Kanner, 2007). This model calculates the pressure created by an ideal stretched spherical rubber shell, and is appropriate for our purpose. The Moody-Rivlin model is defined as follows:

$$P_{elastic} = 2\mu\frac{t_0}{r_0}((\frac{r_0}{r}) - (\frac{r_0}{r})^7)(1 + (\frac{1-\alpha}{\alpha})(\frac{r}{r_0})^2) \tag{4.10}$$

where, μ is the shear modulus, α, a constant, is given as $\frac{10}{11}$ within the Moody Rivlin Model, r_0 is the unstretched radius of the balloon, r is the inflated radius, and t_0 is the unstretched thickness.

Now that $P_{elastic}$ has been established, P_{atm} can be found through the US Standard Atmosphere. P_B can be replaced with the ideal gas law so that the equation can be solved for volume, which results in the following equation:

$$\frac{nRT}{V} = P_{atm}(A) + P_{elastic} \tag{4.11}$$

where $P_{atm}(A)$ is a function of atmospheric pressure in relation to altitude. Solving for volume, the equation becomes:

$$V = \frac{nRT}{P_{atm}(A) + P_{elastic}} \tag{4.12}$$

Modeling of the volume component is now complete.

Incorporating the equation for volume, the final buoyancy equation is:

$$F_B = g\rho(A)\frac{nRT}{P_{atm}(A) + P_{elastic}} \tag{4.13}$$

By including the complex workings of the rubber into the volume calculations, a more realistic and practical model can be generated. The omission of the rubber pressure results in models that do not accurately reflect the observations seen in an actual balloon, as will be shown in section 6.2.

5. The Complete Model that Describes the Behavior of a Balloon

In this section, all the factors that were discussed are combined to form a single description of the balloon's behavior in the earth's atmosphere.

To create the model, the original equation that relates all forces is used as the foundation.
The calculations can now continue. Each component was broken down into specific equations in the sections above. These equations were as follows.

For buoyant force:

$$F_B = g\rho(A)\frac{nR(T_{atm} + \Delta T_r + \Delta T_s)}{P_{atm}(A) + P_{elastic}} \tag{5.1}$$

where $\rho(A)$ is the atmospheric density function in relation to altitude, $P_{atm}(A)$ is the atmospheric pressure function in relation to altitude, $P_{elastic}$ is the elastic pressure derived from the Mooney-Rivlin model for expanding spherical rubber shells, n is the moles of gas in the balloon, R is the ideal gas constant, $T_{atm}(A)$ is the atmospheric temperature in relation to altitude, ΔT_r is the temperature change due to radiation, ΔT_s is the seasonal temperature variation, and g is the gravitational acceleration.

$\rho(A)$, $P_{atm}(A)$, and $T_{atm}(A)$ are functions with respect to altitude, and can be found from the US Standard Atmosphere (NOAA, 1976). $P_{elastic}$ was found and discussed in section 4.3. n must be measured directly with a mass flowmeter, R is a fundamental constant defined as 8.314, g is $9.8m/s^2$, and ΔT_s and ΔT_r were discussed in section 4.2.1 and 4.2.2, respectively.

For gravitational force:

$$F_g = mg \tag{5.2}$$

where m is the mass of the balloon and payloads and g is gravitational acceleration.

For drag force:

$$F_D = \frac{1}{2}d(A)v^2c(R(A))\pi r(A)^2 \tag{5.3}$$

where $d(A)$ is the density function with respect to altitude, $R(A)$ is the Reynolds number function with respect to altitude, $c(R)$ is the drag coefficient with respect to the Reynolds number, v^2 is the velocity of the balloon, and $r(A)$ is the radius of the balloon with respect to altitude.

As before, density and the Reynolds number is found in the US Standard Atmosphere (NOAA, 1976). The relationship between the drag coefficient and the Reynolds number is found in results from experimental testing, and figure 3 was included to show this relationship. It was also determined that this drag force could be used to determine the velocity of the balloon at any altitude by considering the lift force and drag force to be in equilibrium, and solving for v, or velocity. This method is used in section 6.2 to compare the model with collected experimental data.

The final equation for vertical movement of the balloon becomes,

$$\Sigma F = g\rho(A)\frac{nR(T_{atm} + \Delta T_r + \Delta T_s)}{P_{atm}(A) + P_{elastic}} - mg = \frac{1}{2}d(A)v^2c(R(A))\pi r(A)^2 \tag{5.4}$$

Combined with the HYSPLIT model provided by NOAA (NOAA, 2014), a comprehensive model for the vertical and horizontal behavior of a balloon in the earth's atmosphere is found.

6. Experiments and Analysis

In this section, the model found through theoretical analysis is compared with actual data collected from a weather balloon to determine the accuracy of the model. In this comparison, only the shape of the graph is considered; translation of the graph may occur due to differences in temperature between the US Standard Atmosphere Data and the actual on-site data.

6.1. Data Collection

The data used in this comparison was collected from the weather balloon flight FAST-12 by an amateur ballooning group on August 4th, 2013 Maxham et al. (2014). The balloon was launched from Mesquite, NV, at 9:29 am. The balloon was filled with $1.741m^3$ at Standard Temperature and Pressure with Hydrogen, with a molecular mass of $1.00794u$ (Haynes, 2013). The balloon weighed $1616g$, while the payloads weighed $189g$. A GPS module on the balloon transmitted altitude and location data every 30 seconds through the APRS (Automatic Packet Reporting System) amateur radio network, and it remained functional throughout the flight. Figure 6 shows the altitude vs time plot for this flight.
Ascent velocity can be found by finding the derivative of this graph through MATLAB. Then, by replacing time with altitude, the relationship between ascent velocity and altitude can be graphed

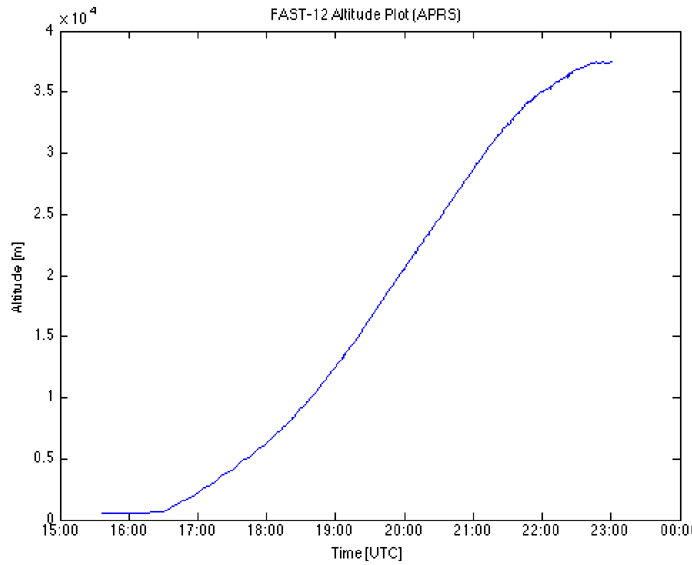

Figure 6: Altitude (m) vs time (s) of FAST-12

as shown in figure 7.
This relationship is required for accurate comparison because the modeling program also gives its results in ascent velocity vs altitude. Although it also gives other results, such as lift force vs altitude, this relationship is the most appropriate for comparison because it can be derived directly from flight data. Results such as lift force vs altitude are harder to compare with flight data, as it requires more data processing.

6.2. Ascent Modeling

A MATLAB program calculates the lift force for altitudes from 100 m to 33000m in intervals of 50m, using equation 5.4 generated in 5. It then calculates the vertical velocity of the balloon using the relationship between the drag force and lift force described in section 3.2.

Two models were considered: the Mooney Rivlin model (which includes elastic pressure) and the non-restoring model. These considerations are shown in figure 8. There is a signficant difference between the Mooney Rivlin model and the non-restoring model. The lift velocity of the Mooney Rivlin model decreases towards the end of the flight, while the lift velocity for the non-restoring model continues to increase.

Now that the two graphs have been found, they can be compared. It is clear that the shapes of the data from flight FAST-12 and the Mooney-Rivlin model are very similar, indicating that the model is quite accurate in its considerations of the factors. However, the graph for the model is translated vertically; this may be due to differences in temperature, altitude, or other factors. Ultimately, it can be determined that the model developed in section 5 is accurate in predicting

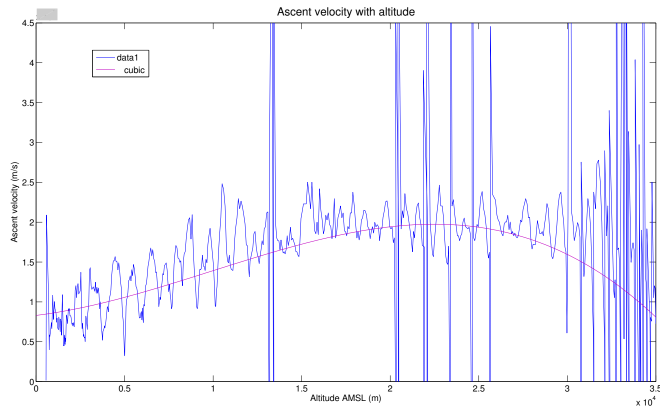

Figure 7: Ascent Velocity (m/s) vs Altitude (m) of FAST-12

the behavior of a balloon.

7. Conclusion

In this essay, I sought to provide a comprehensive scientific explanation of a High Altitude Balloon's behavior in the Earth's atmosphere. I considered the factors of wind, atmospheric temperature, solar radiation, atmospheric pressure, buoyancy, rubber elasticity, and drag and described them mathematically through a combination of fundamental and complex physics concepts. Then, by analyzing the relationship between these factors, I was able to combine these equations into a single model for vertical motion. Additionally, through the use of the HYSPLIT model, I was able to predict the horizontal trajectory of a particle in the atmosphere. By combining my understanding of the balloon's horizontal and vertical motion, I produced a complete model for the balloon's three dimensional motion throughout the atmosphere. Finally, through comparison with experimental altitude data collected by an actual scientific weather balloon, I was able to confirm the accuracy of this model and confirm its applicability to uses in the real world.

Some limitations existed in this essay. First, the dynamic quality of nature means that a completely accurate model is near impossible; the model found in this essay is largely based on theory and assumptions, and although there were attempts to include variations in nature, these attempts were most likely not adequate. This could be improved by using directly measured data from the NOAA or LIDAR satellites, but this is impractical and very expensive. Within the scope of this essay, relying on theory provides adequate accuracy for the model to be useful. Second, the Moody-Rivlin model describing the pressure created by rubber is not the only model. The Gent model also provides a method to calculate the pressure of a rubber shell. There are distinct differences between these two models; however, using both models and comparing them would have been time-consuming and too complicated for this essay. Perhaps, in a future development, the Gent model could be further explored.

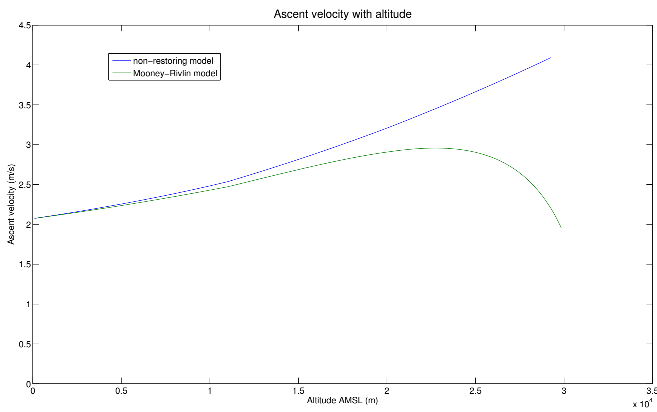

Figure 8: Ascent Velocity (m/s) vs Altitude (m) of the Moody-Rivlin model (described in section 4.3(green) and non-restoring model (blue)

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A. Sample Raw Data from FAST-12

```
Date/Time in serial date format, altitude in meters
41490.67233,562.97
41490.6876,585.83
41490.68795,658.98
41490.68829,708.05
41490.68863,744.02
41490.68898,758.04
41490.68934,779.07
41490.68969,798.88
41490.69039,840.94
41490.69073,864.11
```

⋮

```
41490.85043,23918.88
41490.85078,23980.75
41490.85112,24042.93
41490.85147,24107.85
41490.85182,24177.96
41490.85216,24247.75
41490.85286,24384
41490.85321,24451.06
41490.85355,24516.89
41490.85425,24647.04
41490.85448,24583.03
```

⋮

```
41490.98689,37602.57
41490.98723,37587.63
41490.98758,37575.44
41490.98828,37538.56
41490.98897,37496.5
41490.98932,37489.49
41490.98966,37491.62
41490.99036,37525.45
41490.99052,37503.51
41490.99071,37552.58
41490.9914,37587.63
```

B. Sample Code from MATLAB model

```
% balloon parameters
burst=2.9; % burst radius (m)
mb=1.616; % balloon mass (kg)
mp=0.189; % payload mass (kg)
M=1.00794; % molecular mass of gas (g)

% range of altitudes to sweep (m)
h=[100:50:33000];
M=1.00794; % molecular mass of gas (g)

[rho,a,T,p]=stdatmo(h,0,'SI',true);
rubberrho=1100; % density of rubber (kgm-3)

%Volume Calculations
STPV=1.74149;
LaunchV=(T(1)*101300*STPV)/(293.2*p(1)); %volume of gas at launch (m^3)
initial=((3*LaunchV)/(4*pi))^(1/3);
r0=initial;

d0=mb/(4*pi*r0^2*rubberrho); % uninflated thickness
m=mb+mp; %gravity

% determine n corresponding to initial radius
n=moles(p(1),T(1),initial,r0,d0,LaunchV);
n1=moles1(p(1),T(1),LaunchV);

r1=radius_nonrestoring(n1,p,T);
i1=find(r1>burst,1)-1;
h1=h(i1);
p1=gasp(n1,r1,T)-p;
l1=lift(n1,M,r1,rho,m);
v1=terminalvelocity(n,M,r1,rho,m);

r2=radius_mooneyrivlin(n,p,T,r0,d0,initial);
i2=find(r2>burst,1)-1;
h2=h(i2);
p2=gasp(n,r2,T)-p;
l2=lift(n,M,r2,rho,m);
v2=terminalvelocity(n,M,r2,rho,m);

figure(1);
pl=plot(h(1:i1),v1(1:i1),h(1:i2),v2(1:i2));
title('Ascent velocity with altitude');
legend('non-restoring model','Mooney-Rivlin model','location','northwest');
xlabel('Altitude AMSL (m)');
ylabel('Ascent velocity (m/s)');
axis([0 35000 0 4.5])
```